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# SHORTER COMMUNICATION

## DIMENSIONAL ANALYSIS OF BURNOUT HEAT TRANSFER

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In his book on *Heat Transfer in Condensation and Boiling*, Kutateladze [1] states that experimental results on burn-out heat transfer rates can be correlated by the expression

$$\dot{q}_c = (0{\cdot}16 \pm 0{\cdot}03)\, L \,.\, \rho_v^{1/2} \,.\, g^{1/2}\, \{S \,.\, (\rho_l - \rho_v)\}^{1/4} \qquad (1)$$

where

- $\dot{q}_c$, is the burn-out heat transfer rate, cal/cm² s;
- $L$, is the latent heat of the liquid, cal/g, which is assumed to be at its saturation temperature;
- $\rho_l$, $\rho_v$, are the densities of liquid and vapour, g/cm³;
- $S$, is the surface tension of the liquid, dyn/cm.

It is not clear from the text whether the symbol $g$ represents $g_g$, the local gravitational acceleration (cm/s²), or whether it stands for $g_0$, the conversion constant from force to mass units (= 981 g cm/s² g force = 1 g cm/s² dyn) which appears in equations of motion. Although irrelevant to most terrestrial experiments this distinction could be important under space flight or orbital conditions. A dimensional check shows that the correct form of the equation, taking this distinction into account, is

$$\dot{q}_c = (0{\cdot}16 \pm 0{\cdot}03)\, L \,.\, \rho_v^{1/2} \,.\, \{g_g \,.\, g_0 \,.\, S \,.\, (\rho_l - \rho_v)\}^{1/4}. \qquad (1a)$$

This formula was brought to our attention by a private communication from Professor D. B. Spalding, who grouped the variables into two dimensionless parameters by introducing the mean bubble size as a reference length. The following note is an attempt to give the simplest possible physical explanation of the form of the equation and the size of the experimental constant. A rather more detailed explanation, in terms of the frequency of bubble shedding and the instability of two-phase flow has been given by Zuber [2].

Consider the stability of a growing bubble on a flat horizontal heating surface. Suppose that at any moment the volume of the bubble is $V$ (cm³) and the radius of the circle of contact $r$, the angle of contact with the surface being $\theta$. Then the net buoyancy force upwards on the bubble is $V.\,(\rho_l - \rho_v) \,.\, g_g/g_0$ and the resolved part of the surface tension force downwards is $2\pi r \,.\, S \,.\, \sin\theta$: since the movement of the centre of gravity of the bubble is negligible as long as it remains attached to the surface, these forces must be in static equilibrium.

As the liquid is heated and the bubble of vapour grows, the buoyancy force, which varies as the volume, must increase faster than the surface tension force, $2\pi r \,.\, S$, so that, to preserve static equilibrium, $\sin\theta$ must increase. The limit is reached when $\sin\theta = 1$. If it is assumed that the bubble is then hemispherical the equation of static equilibrium yields

$$2\pi r \,.\, S = \tfrac{2}{3}\pi r^3 \,.\, (\rho_l - \rho_v) \,.\, g_g/g_0. \qquad (2)$$

Any further increase in size makes the system unstable and the bubble is accelerated away from the surface.

Thus the critical size of bubble, $r_c$ (cm), is obtained from equation (2) as

$$r_c^2 = 3S \,.\, g_0/(\rho_l - \rho_v) \,.\, g_g. \qquad (3)$$

Burn-out occurs when the whole surface is covered with vapour—that is when the bubbles crowd on each other so fast that there is no room for fresh liquid to reach the surface and keep it cool. Thus to avoid burn-out one bubble must be able to accelerate away from the surface faster than a new one grows into the space it vacates. In the limiting case we may suppose that the bubbles just touch at the moment of release of the second one.

Let us assume that the bubble maintains its hemispherical shape (any other assumption would only introduce a small numerical correction factor) and that the drag is negligible at the small dispatch velocities involved. Then in the time $t$ (s) taken for one bubble to form, the centre of gravity of the preceding bubble must have moved upwards a distance $r_c$.

Thus

$$r_c = \tfrac{1}{2} a \,.\, t^2 \qquad (4)$$

where $a$ (cm/s²) is the acceleration of the centre of gravity of the bubble.

Once the bubble is launched, buoyancy is the sole force acting on it, so its equation of motion is

$$\tfrac{2}{3}\pi r^3 \,.\, (\rho_l - \rho_v) \,.\, g_g = \tfrac{2}{3}\pi r^3 \,.\, \rho_v \,.\, a.$$

That is,

$$a = g_g \,.\, (\rho_l - \rho_v)/\rho_v. \qquad (5)$$

If $L$ (cal/g) is the latent heat of the liquid and $\dot{q}_c$ (cal/cm² s) the mean rate of heat transfer at burn-out, the time taken to produce a bubble of mass $\frac{4}{3}\pi r_c^3 \,.\, \rho_v$ is given by the equation

$$t = \tfrac{4}{3}\pi r_c^3 \,.\, \rho_v \,.\, L/\dot{q}_c \,.\, \pi r_c^2$$

$$= \tfrac{4}{3} r_c \,.\, \rho_v \,.\, L/\dot{q}_c. \qquad (6)$$

Substituting into equation (4) from equations (5) and (6)

$$r_c = \tfrac{1}{2} g_g \,.\, \frac{(\rho_l - \rho_v)}{\rho_v} \,.\, \left(\frac{2r_c \,.\, \rho_v \,.\, L}{3\dot{q}_c}\right)^2$$

therefore

$$\dot{q}_c^2 = \tfrac{2}{9} g_g \,.\, (\rho_l - \rho_v) \,.\, \rho_v \,.\, L^2 \,.\, r_c.$$

Substituting for $r_c$ from equation (3)

$$\dot{q}_c^2 = \tfrac{2}{9} g_g \,.\, (\rho_l - \rho_v) \,.\, \rho_v \,.\, L^2 \sqrt{[3S \,.\, g_0/(\rho_l - \rho_v) \,.\, g_g]}$$

therefore

$$\dot{q}_c = 0{\cdot}62\, L \,.\, \rho_v^{1/2} \{g_g \,.\, g_0 \,.\, S \,.\, (\rho_l - \rho_v)\}^{1/4}. \qquad (7)$$

Comparing equations (7) and (1a) it is evident that their form is identical, but the constant in equation (7) is about three or four times as large as that determined experimentally. There are several possible reasons for this.

(1) The minimum clearance between successive bubbles must be greater than the limit assumed, in order to allow for deformation of the bubbles, access of fresh liquid, etc.

(2) The bubbles cover an area less than the total superficial heating area, so their rate of growth for a given mean $\dot{q}_c$ is greater than would be calculated if $\dot{q}_c$ were uniform. Even if the bubbles were regularly spaced on hexagonal centres this would reduce $\dot{q}_c$ by a factor of three-quarters.

(3) In many burn-out tests the heating surface is not horizontal, so the acceleration rate normal to the surface will be less than the vertical buoyant accleration.

A further check on the applicability of equation (3) is obtained by substituting typical values for the physical variables to obtain $r_c$.

For water at 100°C, on the earth's surface,

$S$ = 58·8 dyn/cm,
$g_0$ = 1 g cm/s$^2$ . dyn,
$g_g$ = 981 cm/s$^2$,
$\rho_l$ = 0·958 g/cm$^3$,
$\rho_v$ = 0·0006 g/cm$^3$,

therefore

$$r_c = \sqrt{(3 \times 58{\cdot}8/0{\cdot}957 \times 981)} = 0{\cdot}43 \text{ cm}.$$

Although this figure would seem to be a little on the high side it is certainly within an order of magnitude of that observed in practice.

Summarizing, then, we would suggest that a possible physical explanation of the observed empirical law is that bubbles break away from the surface as soon as the buoyancy forces are greater than the surface tension forces holding them on; and that burn-out occurs when the buoyant acceleration of the bubbles away from the surface is no longer great enough to keep the bubbles clear of their successors.

## REFERENCES


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2. N. Zuber, On the stability of boiling heat transfer, *Trans. Amer. Soc. Mech. Engrs*, **80**, 711 (1958).